RANDON PROCESSES

a sample space of a giver

Random variables →

mapping from

дадисл

Experiment into ser

a complex nos.

Discrete time R.P →

the net

A disiste teme

g real

mapping from a sample space or int

distuste time rignal x(^).

time R.P is a collection & ensemble

g

25

Vignal

e.

Ex: 20thing of a fair dis. Ler the outcome be aligned to a

RV A. 12A ≤6 with equal peob.

$$x(r) = A \cos(nwo)$$

a RP is created eith that Comish

With

an entremble

depeier + equally propable discrete time rignals.

Ex flipping a fair coin.

$$x(N = \{-\backslash$$

& x

Heads is

-1

Pails

Hipped in jopped

random

XW) is D.T.R.P

Comistery of

a

the Coin at timen, in no

If the flip of the

coin

the

Oukom

known as

the coin ar

any

the jeep

of

Bernoulli process

Ex Criver a R.P x(^), other

aд

Six

requencof 1st

-is.

way

affects

the

other time, then x() is

R.P I(^), other processes may be generated by transparming 2(^). A usepel transpormation in linear filtering.

Criver

a

Bernoulli

by

filtery

IG). a new process may

process x G).

filtering xlm) with peos order

recursive filter,

(n

.

y(a) = 0.5y

(mai) +xG)

Ex Roulette wheel,

ancy

no.

The

generated

in the

internal [0,1] in

equally

likely to oaur. If the no. obtained

with the

Aurigned to

the

wheel in

Expansi

on

за

x

to the R.UD,

span of the

ses the expirite

to may

x = {x) ^^, where xC) is either

620

to 2010 a one for no

Gud

fo a particular value of s.

may r=no,

ralue x (mo) is a R.U that is

an

defin

ed

The ignal

The

on the Apa sample

lach co eл

here in a

space as for each

valin

g

2 (20).

Henco a R.P may

correspondi

ng

also

he wewed an an

indend

Y (-2), x(-), x(o), x(·), x(2), ..

repents

of

RV,

Each Ruin a

requen

co

has underlying prob.

destitution

div.

Fxem) (d): Px

Ex(m) <d]

+ prob. demita

pencion

fx (a)

(d).

d

(3)

Fxmm

(α),

do

The

joire

Fxcw, x

(mz), . x

(mm)

distibution

pencions

(d... da). Pxfx (a) & d., . . .

x (ox) ≤ da),

Canide a RP peroned from a

requers jo Cranian R.V

26).

Yh

If to R.v xa)

are

known as

sqance.

unconcated, then the

рлол

и

white Camerian Noise

Ensemble

Averages

na vey

random poiseline

Since a Discrete time R.P is an indexed

P.v, we

may

calculate the

mean

g

each

vaiables & generate the

deterministic

mx(n) = [8x (n)] .

known as the mean

value

the process

t

the

process.

as a

g

requence

of

there random

sequence .

This

defines

defines the

average

peretion on.

2

Compu
ting

defines
the

62 (n) = &{|
x (n) - mx (^)/2]
6x

vacionce

squared deviation

E

the
the variance

g

each R.U in the requerce,

process. This represents the
average

the

process away from the mean.

The autocovariance is defined

4

as

$$Cx (x,e) = \{[x(x) - mx (x)] [xco) - m2 (o)]^*]$$

the autocurrelation

relating

$$Six (K, 8) = E\{x(x) x^* (1)]$$

the R.U x(x) & x(^).

of Kil, the autocovariance reduces to variance

$Cx\ (K,K) = 6x$
(x)

The autocovariance & autocorrelation functions

$Ck\ (kil) = rx\ (k,e) - my\ (k)\ m, c)$

For Zero mean

флосии,

are related

by

autocovariance & autocarelation are

equal.

En the discusions, untus states otherwise R.P will

always be assured to have zero

4 autocarelatiei

sequences

may

mear so

that autocovariance

be used

interchangeably.

This assumption results in no

lais for any prouess x(r) that
has

of writegnity
rince,

nonzero mean, a
zero mear

prous yea) may always be
formed by subtracting
from x4)
mear

y(n) = x (m) - My (n)

the

of CK (K, 1)=0 por XII, the R-V x (K) &
x(1) are uncorrelated

+ knowledge
of

using
the
other
one
not help in the estimation

does a linear estimator.

Ex

Harmonic Process

& ponal.

Ex::

An important R.P found in applo.

such as jadar

is the harmonic procers.

да

signal

processing is

real valued R.P is the random phase

sinusoid,

x (^) = A Sin (nwo+

$)

A + wo - fixed constant.

$ -RU

unipormly

unepormly destibuted over the

interval -π to π (ie) prob. density
pene. for & is

The

using

mean

Ma (n)

f(x) =

{

ŋ́

=

271-7

-πT ≤ 2 ≤ T

u, otherwise

the process,

$

= [xml]

= E {A Sin (nwo
+ 4)].

the equation [&{y} = {{gray.
I glad fx ladder.]

TI

m2 (n) = √ A Sin (nwo+ & fp (*)
doc = SI A Sin (not da

Thus 26 is a

26) is

20

x(^) is a zero mean prouess. The
autocorelation

४

91x (x, 0) = = {x(x) x* (8)] = = {{ A Sin
(kwo+ 6) A Sin (Ivory)

[2 Sun ASinB = Cos
(A-15) - (os (A+B)]

2

A

2

This i

expectation

Constant

f

({[Cos (re)co+2x]) .

[[Coo{[(keino +20)] de

+2d

1

2

TT

TT

2 [Sin [imower +27] -

Sin[(render. 2.]).

тап

2 Sr((x+1)w] (and +

Con[(Ken)vo] Sin T

-18

co

-

(SAN&T=0]

Cos [+1)wo]

Sud

rx (k,e) =

नर

Cos [(x-1)

wo]

२

Comple

x

harmoni

proces

a similar

fashion, for the

them

mear

x(n) = Apj

(nwo+*y*)

Mx (n) = E

f the process in

{ACI(OH

O+)]=

E

A

the autocorrelation in

2x (x.1): & {1(k) x* (*)] =

ε[A @j (nur)

for both

functions

Axe-j(n+y)

mean is a

Constant & the

autorevelation rack.1) is a func. off the diss.

b/w kde

Thuis he

2x (k, 2) = rx (x-1,0).

pie

st

second order statistin does not

depend upon

absolute time

process

The autocovariance & autoconsetation sequences provides

imps about the statistical relationship b/w two R.V that are

same proces eg: x(k) & x(x).

derived

the same

from On apple,

envolving

to determine the

Places x (K) +

covariance

more than one R.P it is need

O correlation blu R.0 in

another

yes.

+R.U in another

Cuver two R.P 2 (n) & y(e), the

loss-covariance

defined

by

$$C_{xy}(4.8) = = \{$$

$$[x(x) - \_mx(4)][y(p) - my()) "\}$$

E

of the Leoss-correlation

E

by

$$day(K, M) =$$

$$\& \{x(x) = (0)].$$

яxy

There peretien satripes

the relation.

$$C_{xy}(k, 1) = Say$$

(x.1) - m (a) og (1)

rxy my2

Two R.P x(N)

+y (^)

are said to be uncorrelated

are said to be a

thogonal if

Czy (x,

1)=0.

(Two RP xG)

+y(^)

day

(x-1)=0

xy

Orthog

onal

R.P Ale

not

Zero mean

RP that are

necerravity achogonal

one

if

uncorrelated, bur

are unconclated.

Ex Comider the pair of processes, x(n)

+ y(x) where

yen) = x

(n-1)

The cross conclation b/w x (m) & yo) is

2zy (KJ) = = { (x (k) g+ (1)]) = =

(x(x) x (6-1)] = 2x (x,1-1)

If

yes

The Conu.

g

x ( )

cos a

deterministic

regueres

)

hr

y ( ) : Σ (m) x (n-m)

h(mx

Mind

The cross conclation in

(k,e)

E

Day (41) = 0 (x (H) x +

(1)]. (G20) Σh (m) x

(l_m)]

En

many

Σht (m) rx (k,

d-m)

M:-2

apple, noire is modeled as

additive

9

no

that

4

xG) denotes the signal & WWD the

signal & was the noise, the recorded rignal is

y(n)= x ()+w (n)

The autocorrelation

of you by (1,4) =

E & gensy + (25)

E

is

= [{[x(x) +w(x)]

[[((α) + works]*]

E

= = {x(x)=()] += {x(x) 20+ [1] += {wak)

rus] +&{w(K)=(e)],

of

XG) & Woln) are

uncorrelated, then

((x(kw* (1)] = =
{wreset()]=0

Ry (x,1) = 2x (x,e) +
&w (k,x)

(K,p).
)

Prope
rty

f

tws R. P x G)
+g (n)

are unconclated, then the

g

Sum

autocorelation

is

еди
я

the

2G1=2(^)+y(n)

Д

to the sum

dz (K, l) = Ax (K, d) + by

(x, x)

the autorevelations

of xan &

gas.

are said to be

functi

a

g

fx (x)=

Gaussian

Process

With x = [x, , X., ... In] a

vector

real valued R.U,

8

xin said to be Cranesion Random vecte & he R.U Xi

jointly

the n Rux

Caussian if the joins

peob.density

人

(2πT) ^/2

1G11/2

exp

•{-2(x-mx)" (x"

(x-me) }]

where

mx =

[m., me, ... m.] in a vector

co

Contou

contai

ning

mean

8 x

mi = εSxy.

matrix with dementi

(is a symmetur positive definite matin

Cij that

ale

the

Covariances blw Xi +

Cij = = { (xi-m) (x4-m;)].

|Cx) – dekiminent

A discrete time P.P xe in said to be Caussian

g

The covariance matix.

every firète collection

д

samples of I (~) Ale

8

A Caurian P.P in completely defined

Covariance matrix.

Stationary Processe s

are

known.

jointl y

Graueran

Once the mean reches &

mary vignal proving appl., the statistin de

CM

ofter independent

of time.

called

stationary

In

ensemble

averages of

a R.P Are

Such

proceses

Jus

g

time

(ie)

of the

independe

nt of

oder density

pencion of

fxcm) (d) =

fx (n+x) (α)

Process.

a R.P x(n) in

all 14, then

The

proces Stationary.

To a

is said to be

fiest

order

flor

pest

order

pocko oder stationary

stationary

procers, the

statistin coill be independent

Constan

of

$m_x(n) = m_x + 6s\ 6$

A рлоша

The second order

depends

only

if

00

h

+ vacance

time. Ex. The mean coill be

^

is said to be second order statis

join

statis

ary

density punction fxcnis,
Xina) (d, de)

the difference 12-1. &
nor

12-n, & nor on individual

times n. dne. The process and in
second orde statis

k, the

stationar
y

proceses I (n) & x(n+k) have same

if for any x,

second-order of joint
denty perction.

Exc.), x(on) (L.,de) =
Excm++) x (44)

If a
process

be

pes

t

(0.0)

in

second-order

stationary

oder stati

statio

nary

(L., dr)

then is coin also

In addition second order

stationary

have second-order statistin that are vivacious

processes have second-order statistin

to a time ship ja process. The

autoconétation reque

has the

property,

l+n).

rx (ked) = rx (x+n, 1+r),

on he

The Covelation blw RU x(k) & 2 (1) depends only dep. K-1, seperatin two Ro in time. This dip. kla Called the lag,

rx (x, l) = rx (x-e)

A

рлоги

is said to be stationary of order l

L

of the prowess xa) + x(n+k) have the

same 1-th ader

joine

dessity

pinctions.

A

that in

рошо

statioray for all siders

2.70

said to be stationary

in the struct sense.

Wide Sense

Stationarity

A R.P XW) in said to be wide reme

stationary

The

following

1. The mean

condition

Co

satisped

да

a proces in a constant, My G^)= mx

2. The autorevelation rx (K.1) depends only

3. The vacionce

In 2-ard

statuerary

En

w3s

g

process in firite, $(x(o) < \infty$.

prète, $(x(o) < \infty$.

Constraints

Constraints are

So, was in weake

place

d

are

on the diff.,

Kal.

placed in density

perches.

in emenble average.

Constrains than second-order

stationarity.

proces
s

Bur

since

Craursion

leem

mears

&

Corauance

in completely defined
in

css is

let to

equivalen
едить

stiverem staturarity.

f cuss

Ex:

1. Bernoulli

pachuele instáve

not wss

Ости

-&t ouron

process & t

doen nor aper

the surcome

2. Mandon phase

nimesid

2 C^)= A Sin (nwo+).

Sinusoide

proces

X (^)= A Cos (nw .) .

RP at

ay

as other instant.

(Two peores 20) &y() are raid to

be jointly

stationary if xen)

& yen)

are wide surse

stationary

of the cross covelation

Ixy(x,e) depends only

on the

wide

dijs, k-l

Ray (x1)=

2xy

(x+n, 1+n)

xy

.

Szy

Properties

Ray (and) { xcary

= 0)).

autocar clation

Property -

Symmetry

перши

regunce

of

WIS RP

леднел

a

SMMP

The autoconelation

requence

of

Conjugate symmetria

punction ; &,

rx (k) = xx* (-x)

a wss RP in

a

requence in
myownetrz

so a mel

the autocovelation

ром

prouen

,

леднико

Xx (x) = xx(−x).

Propertya –

Mean

ворићс

я

The

Squar

value

autounelativn

2

ледило за это росло

value of the

process, реосец,

wel to the mear

lag

120 ~

equal

squale

22 (0)= = {laca)

1^yo

Property 3 - Maxumien

value

The

magnitude of

lag k

a wiss R.P ar

at

lag

k =0,

the autocavelation

ледшее з

i cepper

2x (0) = 12x(k)!

Properte 4 -

Periodicity

Such

The auto velation

har

some

Xx (ko): 2x (0)

bounded by its

value

wss RP

require ja

coss

Ko, then ex(k) in perode

with period

ko.

E

{ (x(n) =

x(^-K2))"]:0

& xa) i raid to be

mean

одном ренов

6

Ex

xander phan stressid,

6th

Једши

xG). A las (nwo+p), the autoren.

Sequen

dre (x): A2 (on (K().

If

ZA

wo D/N, ther

penedir with perined N + x

G) an

Ex(k) in

penedir

square periodo

Auto covariance &

Autocome latio Matères

The autocovariance & autocorrelation

леднемо

are

mear

00

importar second-order statistical characterrations ofter represented

in matix form.

x = [x(0), x(1), . . .

x(p) JT

is a vector

Z p+1 values

of a proses a(n), the outer

Product

* *

H

x(0) x*(0)

x (0) x* (1)

=

x (0) x* (p)

x(1) ** (%)

x(·) x* (1)

x(1) x*(p)

x(p) x* (0) x (p) **

(1)

x(p) x* (p)

is a

(p+1) x (p+1)

matrix.

of x in

wss, & wing symmetry

property of autocen

2x(x)= 2x* (-x), the autocnelation mateix in

sequenc

e,

E

Rx == [xx] = 9x

(0)

2x (0) Rx (21)

rx (-p)

Яx (i) Я x (o)

xx (1-p)

2x (p)

2x (p-1)

λx (0)

2x (0) 2x*(1)

2x (p)

2x (1) dxlo)

2x (p-1)

7

rx (p)

2x

(p-1)

2x (0)

The autocovariance matrix

is

formed by

fourning

ar outer

The vector (x-mx) with

(x−mx) with

itself

itself + taxis

produser

of expected

value.

C2 == { (x-m2)

(x−m;)"]

F

The relation b/w Rx & Cx

Cx = Rx − m2 ma

H

is

Where

mx = [mx, mx()

~

a чего

mear

processes the

autocovariance & autocorrelation

matives are

equal.

The autocorrelation matix, in addition to berg

bermitial, all

Tearns

along

ๆ

The teens

each

of a Hermitian

Toeplitz Matrix

En cave

the

diagon

als

are

equal.

The Rx in

real

valued random

peorces, the autoconclation

Properties of

matix à a

symmeter Toeplitz mater

Autocorrelation Mater

Property:!

The autocorelation matrix

aза

a Wiss R.P xh) in

a Hermetian Goeplitz matix,

Rx = Soep (2260),

des,... d. (p).

Property : 2

The autocorrelation matrix of a

Was R.P is

be shown than

non negative

deprite, Rx >0.

Proof or mun

a" Rx a> o

matix, ther

Ra

that if tx is an

autocorectations

for any vector a. Since Rx =

E{xx"],

a" Rx a = a" [xx"] a =

&{a"xx"a]

111

=

E{(ax)

(x*a)]

a" he a = =

{1a" x 1"]

sire "x

la" x 1" %o jo

any a [{1a" x

1"]>01

Property

:3

The

eigenvalue

s

,

ues, dx,

of

the autocarelation mater

g

a wiss random

proces

are real &

nonnegative

This

in

becam

4 the

Hermition matrix real

eiger

values f

nonnegative

deprito

mated songatur.

Exampl

e

Autoconclation matix

g

randon phan

sinusoid

the autocanelation repere of

randon phon

перио

Mx (K) = — A2

(on (XWo)

A 2x2 Autocar. matio i

A2

Re: [cove

Rx

Co wo

The

Eigenvalue
s

Caro

Convo

ale

$[Rx - \sqrt{2}) 50$

0

obtained by
solvey,

8

4-d Coswo

Coswo

1-d

J

(1-4)2

Cosco 20

->

1-1 =+ Cons) d = 1- Coswo

7.10

de = 1 + Cosw 1+

7,0,

of the determinant

det (Rx) = 1-Coswo=

5m2wo»,0

Rx = nonegative

deprite of if

Wo #0, TT, Rx =

parti

in

depurat

o.

Example

Conside

g

tus

a complex valued proces comsting of

Complex exponentials,

(nw,

gal = Aejrow +p.)

where A,B,,U2 - Constants

+ nej(nw2+α2)

a sum

uncaneland random variables

The

Autocorrelation

Exponential a(n) = Aej (nwo+)

repence of a single complex

in

rx (x) = 1A)2 ejtwo

The autoconclation requere

& y(a)

by (k) = 1A12 @jkw. +

1A)2 ejkwa

The

autoconctation mativa i

Ry =

1412 /

ماره

&

tejo,

the

y

The

eiger

eiger value

8 by

au.

[Ry - 121

=0

2-1 ej te jur

ميارم

2-2

~) (2-4)-(e) (ette)=0

=) (2-1) 2. I to j(v) ,

1

Hej

a)

co.

+1

te -j (vw) +c) (w1-w2)

-) (2-1) 2 = 2 + ej) .

-)

(2-1) = 2+2600

(w1-w1)

-) (2-4) = 2 (14 (os

(Wi-w1))

2

2 (2 Car

(wide))

(2-1) = 4

CON (W-W2

र

22-1 = ± 9 Cos2

g

-) d=212600

(v)

2) =

+

L

ام

%0

1,0

तो = 2+2000

(1-2) 12=2-260

(021-V2) 8,0

२

1+ las 20:26010